3.3 Division of Polynomials and the Remainder and Factor Theorems

❖ Long Division of Polynomials and the Division Algorithm

Recall: $275 \div 13$

Long Division:

$$\begin{array}{r} \text{Quotient} \\ \text{Divisor}\overline{\big)\ \text{Dividend}} \\ \underline{\qquad\qquad} \\ \text{Remainder} \end{array}$$

The Division Algorithm						
$f(x)$	$=$	$d(x)$	$\cdot$	$q(x)$	$+$	$r(x)$
Dividend		Divisor		Quotient		Remainder

*** If a power of x is missing in either a dividend or a divisor, add that power of x with a coefficient of 0 and then divide. → If there is a missing term, fill in with a 0.

*** If there is a remainder, write the result in the following form:

$$quotient + \frac{remainder}{divisor}$$

Ex. Divide using long division. State the quotient, $q(x)$, and the remainder, $r(x)$.

(a) $(6x^3 + 17x^2 + 27x + 20) \div (3x + 4)$

Check:

$$6x^3 + 17x^2 + 27x + 20 = (3x+4)(\qquad\qquad\qquad)$$

Since the remainder is 0, we can say that $(3x+4)$ is a factor of $6x^3 + 17x^2 + 27x + 20$.

(b) $\dfrac{18x^4+9x^3+3x^2}{3x^2+1}$

(c) $\dfrac{x^3-2x^2-5x+6}{x-3}$

❖ **Dividing Polynomials Using Synthetic Division**

Synthetic Division: the divisor must be of the form $x-c$.

c | Coefficients of the Dividend

Quotient Remainder

***** The degree of the first term of the <u>quotient</u> is <u>one less</u> than the degree of the first term of the <u>dividend</u>.**

*** Synthetic division requires that you divide by the ZERO that corresponds to the factor. $x-c=0$

$x=c$ (ZERO)

Ex. Divide using synthetic division.

(a) $\dfrac{x^3-2x^2-5x+6}{x-3}$

(b) $\left(x^2-6x-6x^3+x^4\right)\div(6+x)$

❖ The Remainder Theorem

The Remainder Theorem

If the polynomial $f(x)$ is divided by $x-c$, then the ***remainder*** is $f(c)$.

Ex. Use synthetic division and the Remainder Theorem to find $f(3)$ where $f(x)=x^3-7x^2+5x-6$.

Ex. (#48) Use the remainder theorem to determine if the given number *c* is a zero of the polynomial.

$g(x)=2x^4+13x^3-10x^2-19x+14$

(a) $c=-2$ (b) $c=-7$

❖ The Factor Theorem

The Factor Theorem

Let $f(x)$ be a polynomial.

a) If $f(c)=0$, then polynomial $x-c$ is a factor of $f(x)$.

b) If $x-c$ is a factor of $f(x)$, then $f(c)=0$.

Ex. (#55) Use the factor theorem to determine if the given binomial is a factor of $f(x) = x^4 + 11x^3 + 41x^2 + 61x + 30$.

(a) $x+5$

(b) $x-2$

Ex. (a) Factor $f(x) = 2x^3 - 3x^2 - 11x + 6$, given that -2 is a zero.

(b) Solve. $2x^3 - 3x^2 - 11x + 6 = 0$

Ex. (#102) (a) Factor $f(x) = x^3 - 3x^2 + 100x - 300$, given that 3 is a zero.

(b) Solve. $x^3 - 3x^2 + 100x - 300 = 0$