3.4 Zeros of Polynomials

In this section, we study methods for finding zeros of polynomial functions.

Remember: the relationship among ***zeros***, ***roots***, and ***x-intercepts***.

The ***zeros*** of a function are the ***roots***, or ***solutions***, of the equation $f(x)=0$.

The ***real zeros***, or ***real roots***, are the ***x-intercepts*** of the graph of *f*.

❖ Rational Zero Theorem

This theorem provides us with a tool that we can use to make a list of all ***possible*** rational zeros of a polynomial function.

The Rational Zero Theorem

If $f(x)=a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ has *integer coefficients* and $\frac{p}{q}$ is a *rational zero* of *f*, then ***p*** is a factor of the constant term, a_0, and ***q*** is a factor of the leading coefficient, a_n.

Find all the possible rational zeros:

1.) ***p***: List all the integers that are factors of the constant term.

2.) ***q***: List all the integers that are factors of the leading coefficient.

3.) Possible rational zeros $= \dfrac{\text{Factors of the constant term}}{\text{Factors of the leading coefficient}} = \dfrac{p}{q}$

Ex. Use the Rational Zero Theorem to list all possible rational zeros for the given function.

$$f(x) = -4x^4 + 5x^3 - 7x^2 - 6x + 8$$

p: ________________________________ (factors of 8)

q: ________________________________ (factors of –4)

$\frac{p}{q}$: ____

Find all real zeros of a polynomial function $f(x)$:

Step 1: Find all possible rational zeros $\left(\pm\frac{p}{q}\right)$.

Step 2: Graph the function and determine which zeros (x-intercepts on the graph) to use in synthetic division.

Step 3: Perform synthetic division to determine which possible zeros yield a remainder of zero. If the degree of a polynomial is 3 or higher, continue to use synthetic division (repeat Step 2 to Step 3) until another zero is found.

Step 4: Rewrite the function as a product of factors, linear and quadratic. Zeros of the quadratic factor are found by factoring, the quadratic formula, or the square root property.

Step 5: Solve $f(x)=0$.

Properties of Roots of Polynomial Equations

1. If a polynomial equation is of degree *n*, then counting multiple roots separately, the equation has *n* roots.
2. If $a+bi$ is a root of a polynomial equation with real coefficients $(b \neq 0)$, then the imaginary number $a-bi$ is also a root. Imaginary roots, if they exist, occur in conjugate pairs.

The Fundamental Theorem of Algebra

If $f(x)$ is a polynomial of degree $n \geq 1$ with complex coefficients, then $f(x)$ has at least one complex zero.

Ex. Find all zeros of $f(x)=2x^4+3x^3-15x^2-32x-12$.

Ex. (#22) Find all zeros of $f(x)=7x^3-x^2-21x+3$.

Ex. (#38) A polynomial $f(x)$ and one or more of its zeros is given.

$$f(x)=2x^5-5x^4-4x^3-22x^2+50x+75;\ -1-2i \text{ and } \frac{5}{2} \text{ are zeros}$$

(a) Find all the zeros.

(b) Factor $f(x)$ as a product of linear factors.

(c) Solve $f(x)=0$ the equation.